Adaptive Method for 2.5D Scattered Point Approximation

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ABSTRACT

In this study, we propose a method for approximating 2.5D scattered points by a B-spline surface. The method generates a B-spline surface for approximation defined by control points that are determined by directly reflecting the errors between the surface and input points. It adaptively refines the control points for efficient error reduction. Examples given in our study show that the proposed method performs better than the existing multilevel B-spline approximation method.

CCS CONCEPTS

• **Computing methodologies → Parametric curve and surface models**;

KEYWORDS

B-spline, 2.5D, Surface, Approximation, Adaptive approximation

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1 INTRODUCTION

3D scanning is considered a primary means of acquiring shape information about 3D objects. A 3D scanner generates depth data that are usually treated as 2.5D scattered points. The points are discrete entities and from them complete geometric information, including the derivative properties, is difficult to obtain.

Fitting the points using a B-spline surface has been conducted in practice. The multilevel B-spline approximation (MBA) method [Lee et al. 1997] and its variants such as [Bracco et al. 2017; Lee et al. 2005; Seo and Chen 2009, 2010; Zhang et al. 1998] are often used in the surface reconstruction of 2.5D points. The MBA method produces a B-spline surface that approximates the points. However, the accuracy of approximation relies on a refinement step, which increases the number of control points by a factor of four. Therefore, the number of control points may grow considerably if a strict tolerance for approximation is considered.

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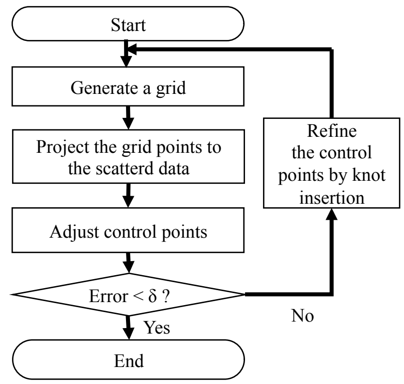

Figure 1: Overall Process of the Proposed Method

In this study, we propose a novel method for approximating 2.5D scattered points by a B-spline surface. The method does not depend on the least squares scheme but determines the control points directly using the approximation error computed by projecting a point from the surface onto an input point set. The area of the largest error is detected, and new control points are generated by knot insertion for local error reduction. This process is repeated until the desired accuracy is achieved. When compared to the MBA method, the proposed method may improve the accuracy of approximation for a similar number of control points and require less computation time for a given tolerance with a large input point set.

2 PROPOSED METHOD

The overall process of the proposed method is given in Figure 1. A set of 2.5D points $\mathbf{p}_d$ is provided as input.

Consider a B-spline surface $\mathbf{r} = (x, y, f(x, y))$. Here, x and y are normalized such that $x, y \in [0, 1]$. The surface is defined with a control net $\Omega = \{(x_k, y_l, \phi_{kl})\}$, $x_k, y_l \in [0, 1]$. We then have:

$$f(x, y) = \sum_{k=0}^{m_k} \sum_{l=0}^{n_l} \phi_{kl} N_{k,4}(x) N_{l,4}(y), \tag{1}$$

where $N_{k,4}(x)$ and $N_{l,4}(y)$ are the B-spline basis functions defined for the knot vectors $\mathbf{T}_k = \{t_0^k, t_1^k, \cdots, t_{m_k+4}^k\}$ and $\mathbf{T}_l = \{t_0^l, t_1^l, \cdots, t_{n_l+4}^l\}$. The input surface is initially given as a plane.

A grid $\mathbf{G} = \{(x_k, y_l, g_{kl})\}$ is created over the surface. Here, the x_k and y_l values of the grid points correspond to the knot values of $\mathbf{T}_k$ and $\mathbf{T}_l$.

The grid points are projected onto the scattered data points. Point projection is performed using the method in [Moon et al. 2017]. We assume that $\mathbf{g}_{kl}^*$ is the projected point of a grid point $\mathbf{g}_{kl} \in \Omega$ in the direction $\mathbf{n}$ onto $\mathbf{p}_d$, $d = 1, \cdots, n$. Then we have

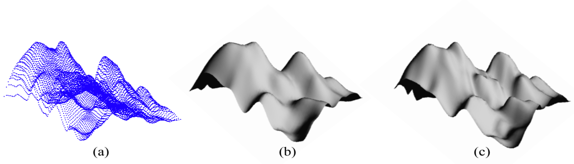

Figure 2: (a) Input data (b) Result by the MBA method (c) Result by the proposed method

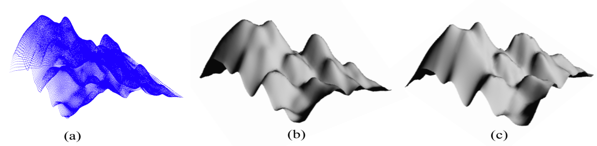

Figure 3: (a) Input data (b) Result by the MBA method (c) Result by the proposed method

$\mathbf{g}^*_{kl} = \mathbf{g}_{kl} + t\mathbf{n}$, where $t = \frac{\lambda - \mathbf{g}_{kl}\cdot\mathbf{n}}{||\mathbf{n}||^2}$ with $\lambda = (\mathbf{c}\cdot\mathbf{n})/c_0$, $\mathbf{c} = (c_1, c_2, c_3)$. c_i $(i = 0, 1, 2, 3)$ are given by

$$c_0 = \sum_{d=1}^{n} \alpha_d, \quad \mathbf{c} = \sum_{d=1}^{n} \alpha_d \mathbf{p}_d$$

with the weight factors $\alpha_d = ||(\mathbf{p}_d - \mathbf{g}_{kl}) \times \mathbf{n}||^{-4}$ for $\mathbf{p}_d$. In this computation, we consider 30 points in the neighborhood of the projection direction to reduce the computation time for projection. Here, $\mathbf{n}$ is chosen to be $(0, 0, 1)$. The neighboring points are efficiently obtained using the kd-tree data structure. Then, $\Delta_{kl} = ||\mathbf{g}_{kl} - \mathbf{g}^*_{kl}||$ becomes the error between $\mathbf{r}$ and $\mathbf{p}_d$.

The surface $\mathbf{r}$ is adjusted by changing ϕ_{kl} because the x and y components of Ω and $\mathbf{G}$ are the same. We then estimate the new control points for approximation by $\phi_{kl} = \phi_{kl} + \beta\Delta_{kl}$. The value of β is determined as follows: The control point ϕ_{kl} to have $\mathbf{r}$ interpolate the point (x_k, y_l, g_{kl}) can be determined by:

$$\phi_{kl} = \beta g_{kl}, \quad \beta = \frac{N_{k,4}(x_k)N_{l,4}(y_l)}{\sum_{a=0}^{m_k}\sum_{b=0}^{n_l}(N_{k,4}(x_k)N_{l,4}(y_l))^2}. \qquad (2)$$

Eq. (2) implies that the amount of change to the control point is directly related to the change in geometric shape. Multiple β values depending on x_k and y_l exist. In this study, we use $\alpha \approx 1.49$, which is the value when $x_k = 1/3$ and $y_l = 1/3$ for a uniform cubic B-spline surface patch with 4×4 control points.

If the adjustment produces an error that is less than the user defined tolerance δ, the process is terminated. Otherwise, the number of control points corresponding to the area of the largest error is adaptively increased by knot insertion. Next, the values of x and y for the grid generation are adjusted to match the x and y coordinates of the grid points with those of the control points by considering both the inserted knot values and the x and y coordinates of the generated control points. Once the coordinates are matched, the process continues from the grid generation step. This entire process is repeated until a surface with the desired accuracy is obtained.

3 EVALUATION

The first example is a simulated terrain represented by 9,801 data points as shown in Figure 2(a). The MBA method approximates the points using a B-spline of 23×23 control points. The root mean square error (RMSE) of the approximation is 0.107539. The proposed method uses 22×22 control points with an RMSE of 0.104229. It takes 0.037s and 0.338s, respectively. However, the proposed method shows more details than does the MBA method with a similar number of control points as shown in Figure 2(c).

Figure 3 (a) shows the same terrain with more points (249,001). The MBA method requires 1.28s to approximate the data using 35×35 control points with an RMSE of 0.046803. By constrast, the proposed method achieves a smaller RMSE of 0.044081 using only 22×22 control points for 1.05s.

4 CONCLUSION

In this study, we propose a method for adaptively approximating 2.5D scattered points. Our method determined the control points directly using the approximation error computed by projecting a point from a B-spline surface onto an input point set. The maximum local error was reduced by adaptive refining the control points through knot insertion. The proposed method performed better than the MBA method.

Currently, an empirical value of the weight is used to adjust the control points for approximation. However, it is expected that an optimal weight estimated by considering the underlying geometric shape may improve the proposed method. This is recommended for future work.

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